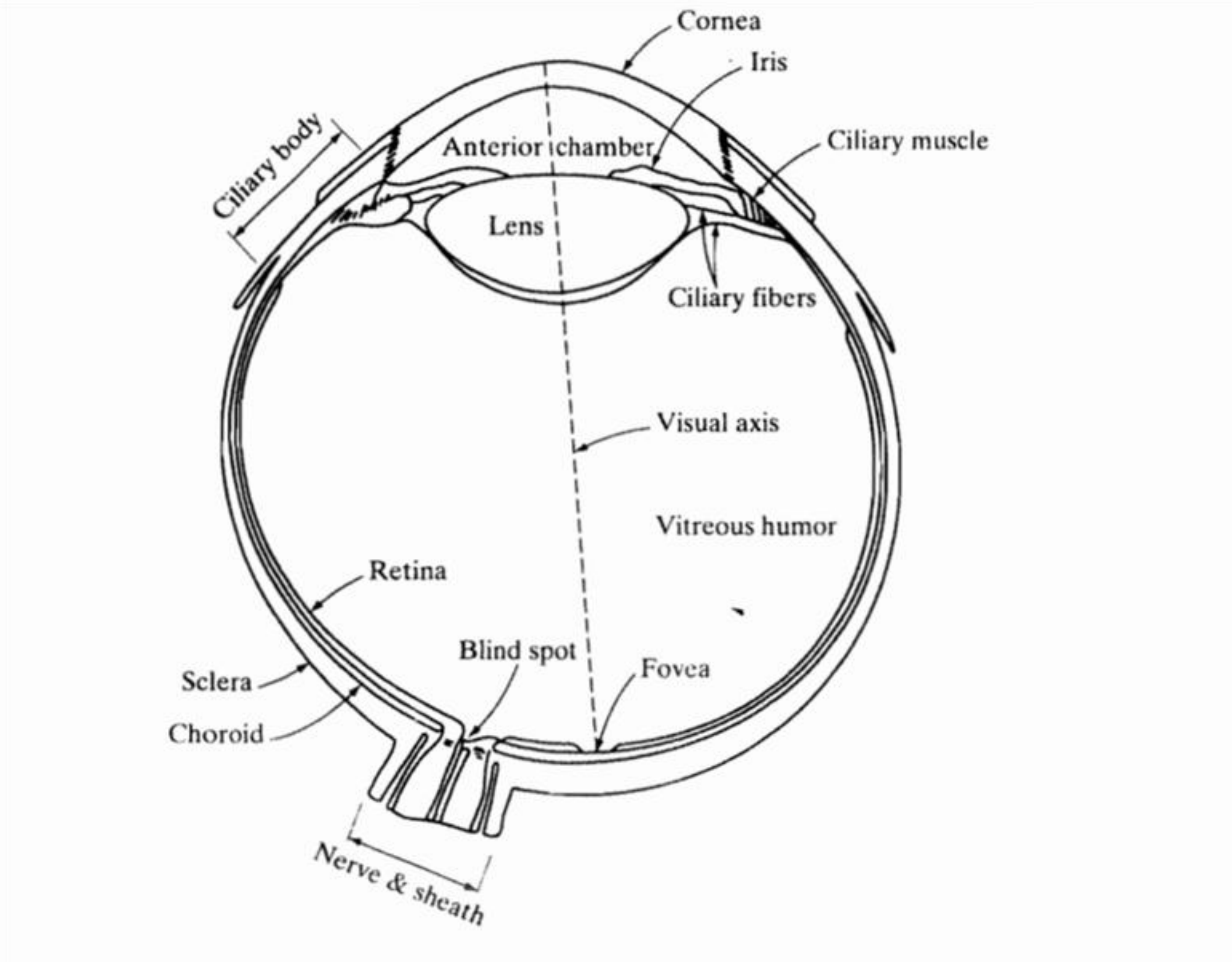
Digital Image Fundamentals

Chapter 2

Structure of Human Eye



- Three membranes enclose the eye

1. The cornea and sclera outer cover
2. The choroid
3. The retina

- There are two types of receptors in retina: cones and rods.

1. Cone vision is called photopic or bright-light vision
2. Rod vision is scotopic or dim-light vision

- Fovea: central portion of the retina and are highly sensitive to color. Cones are located primarily in the central portion of the retina
- The absence of receptors in this area causes the so-called blind spot

Distribution of rods and cones in Retina

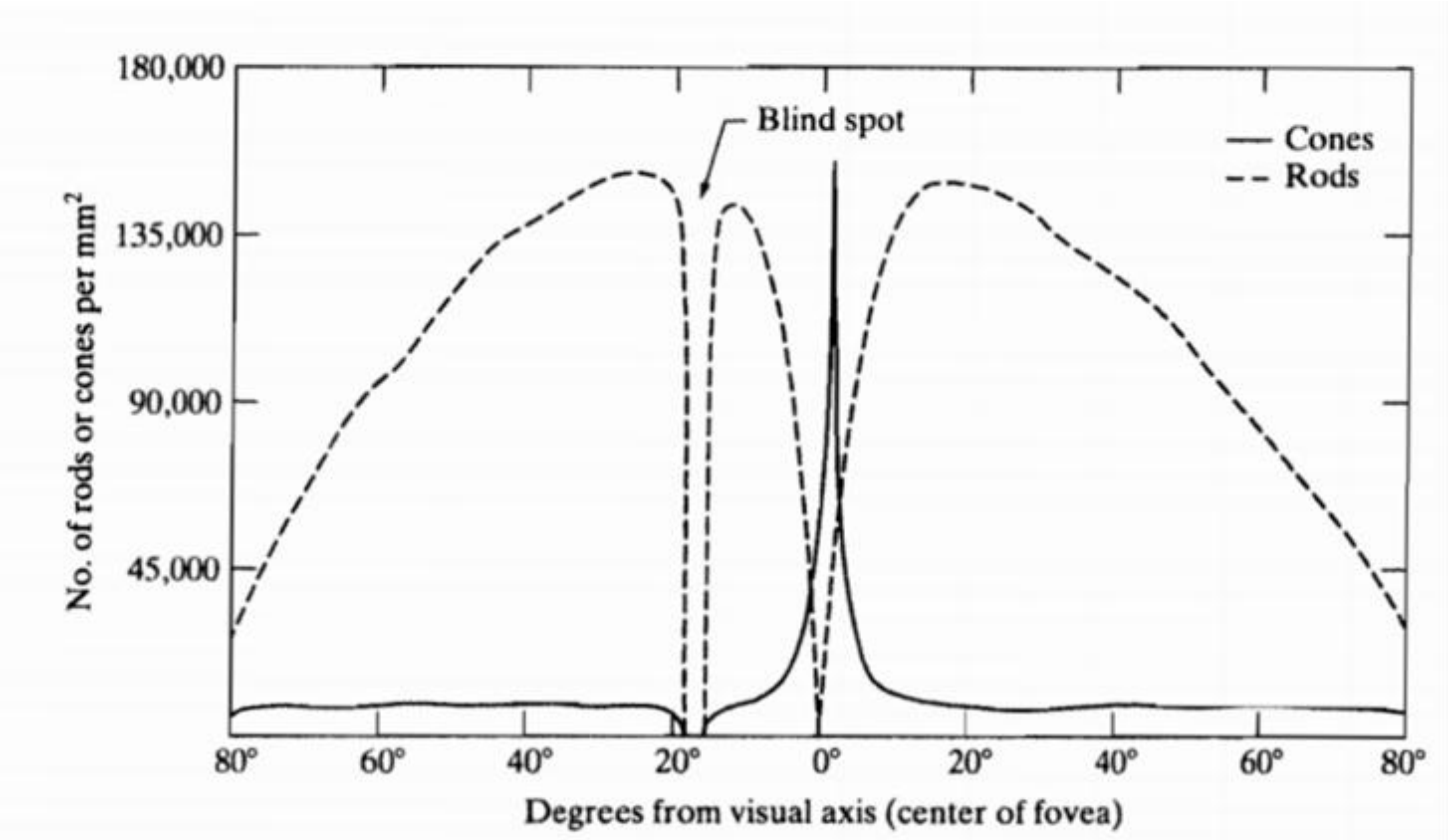
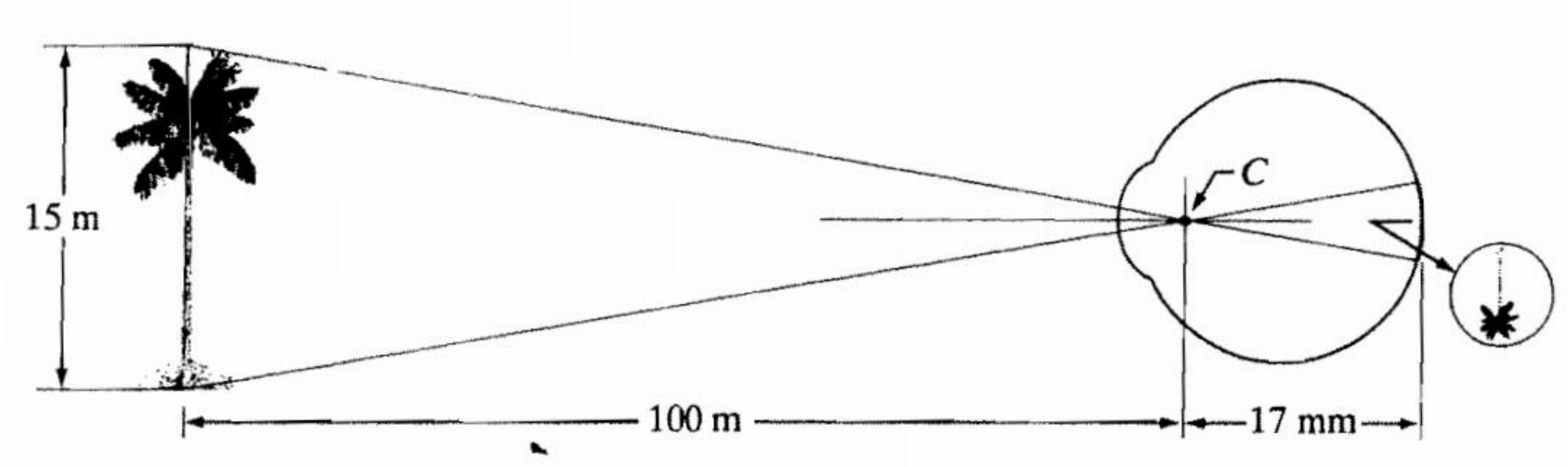


Image Formation in the Eye

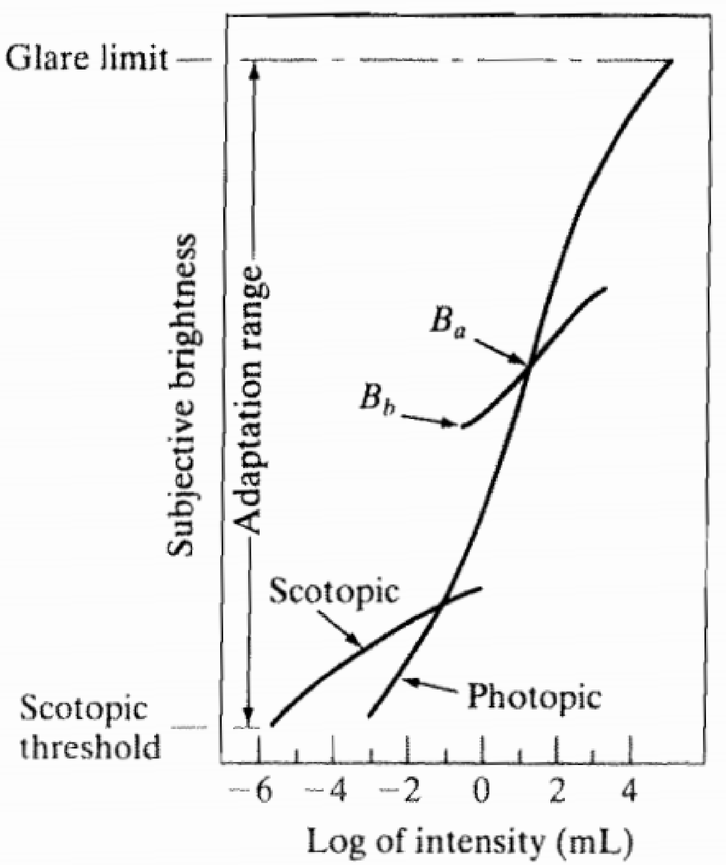


Brightness Adaptation and Discrimination

- Subjective brightness (intensity as perceived by human visual system) is a logarithmic function of the light intensity incident on the eye.

- However the visual system cannot operate over such a range simultaneously, it accomplishes this large variation by changes in its overall sensitivity, a phenomenon known as **brightness** adaptation

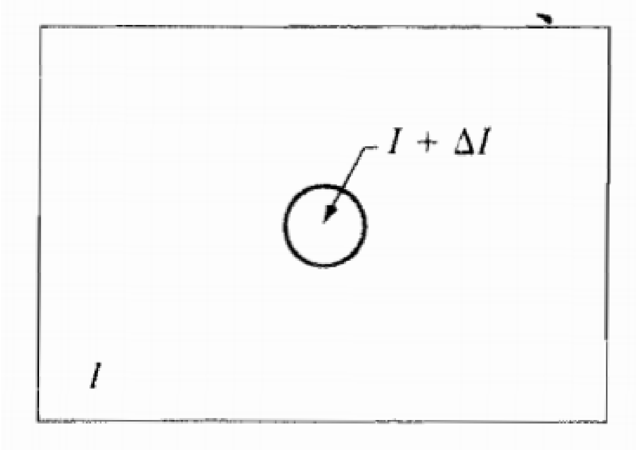
- Short intersecting curve represents the range of subjective brightness the eye can perceived when adapted to the level B_a



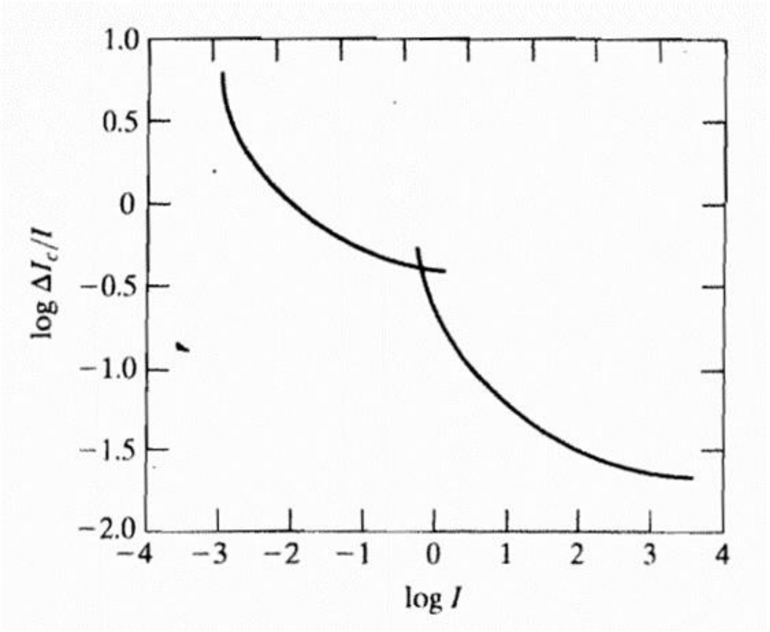
Weber Ratio

- Brightness discrimination is the ability of the eye to discriminate between changes of intensity at any specific adaptation level.

- The quantity $\Delta I_c / I$, where ΔI_c is the increment of illumination discriminable 50% of the time with background illumination I , is called the weber ratio.



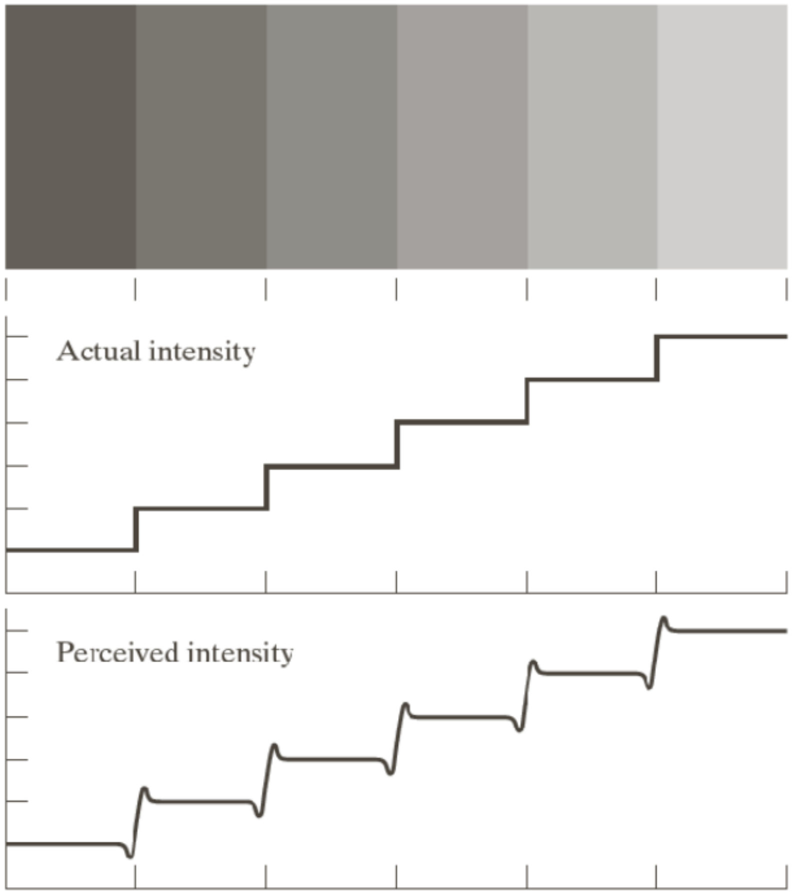
- This curve shows that brightness discrimination is poor (the Weber ratio is large) at low levels of illumination.
- It improves significantly (the Weber ratio decreases) as background illumination increases.
- The two branches in the curve reflect the fact that at low levels of illumination vision is carried out by the rods
- whereas, at high levels, vision is a function of cones



Mach Band Effect

Visual system tens to undershoot or overshoot around boundary of regions of different intensities.

Edges between bars appear brighter on the right side or darker on the left side



Simultaneous contrast

- Perceived brightness does not depend simply on its intensity
- Center square appears to the eye become darker as the background gets lighter

Optical illusion

- The eye fills in nonexisting information or wrongly perceives geometrical properties of objects.

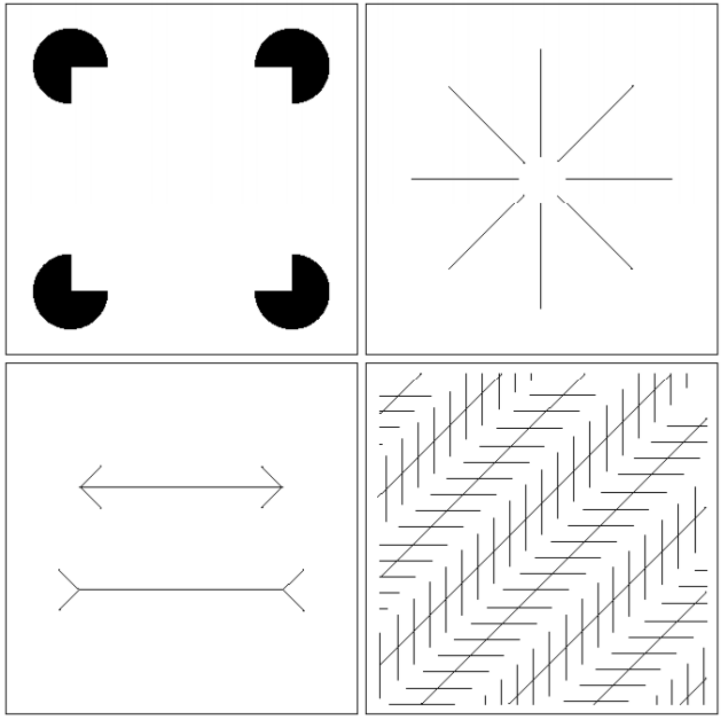
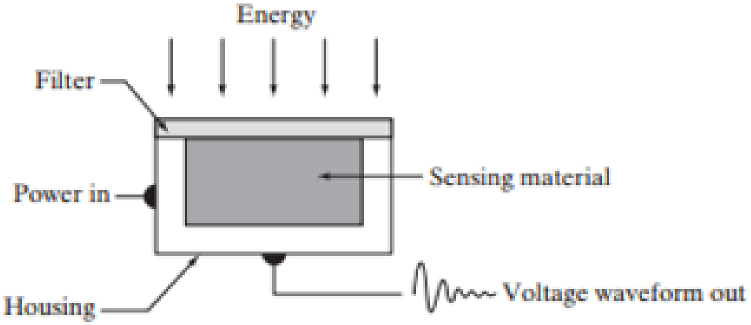
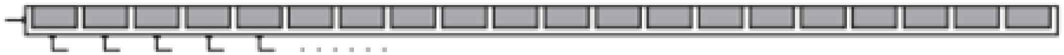
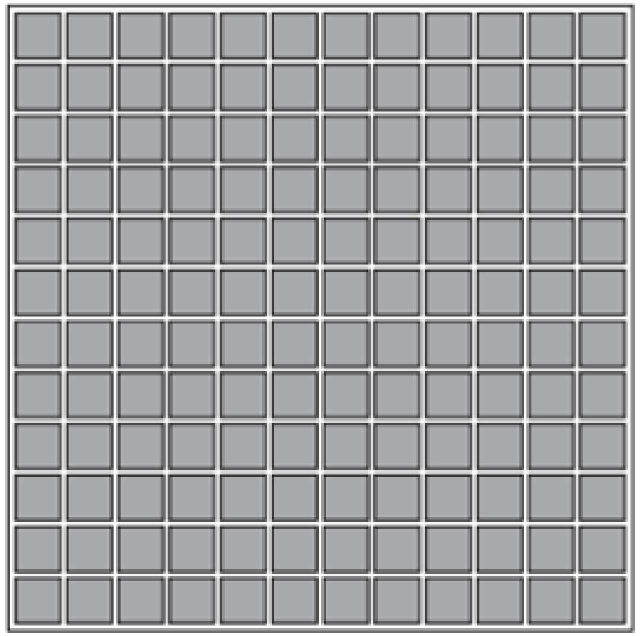


Image Sensing and Acquisition

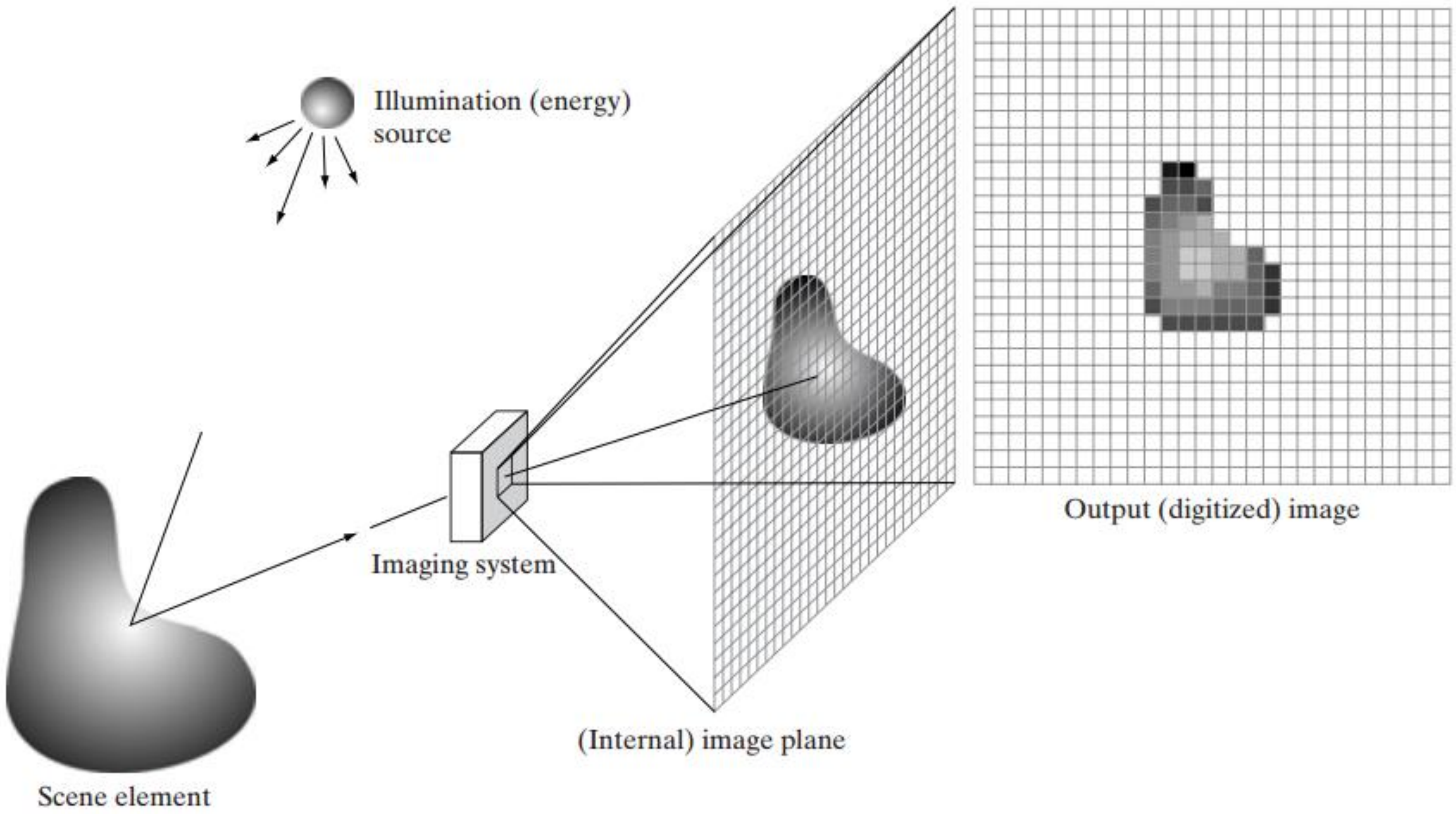






- Incoming energy is transformed into a voltage by a combination of the input electrical power and sensor material that is responsive to the type of energy being detected.
- The output voltage waveform is the response of the sensor, and a digital quantity is obtained by digitizing that response.

Image Acquisition Using Sensor Arrays



A Simple Image Formation Model

- We denote images by two-dimensional functions of the form ***f(x, y)***

$$0 < f(x, y) < \infty.$$

The function f(x, y) may be characterized by two components:

(1) The amount of source illumination incident on the scene being viewed, and

(2) The amount of illumination reflected by the objects in the scene.

These are called illumination and reflectance components and are denoted by ***i(x, y)*** and ***r(x, y)***, respectively.

- $f(x, y) = i(x, y).r(x, y)$

$$0 < i(x, y) < \infty$$

$$0 < r(x, y) < 1$$

The intensity of a image at any coordinates (x_0, y_0) be denoted by

$$\ell = f(x_0, y_0)$$

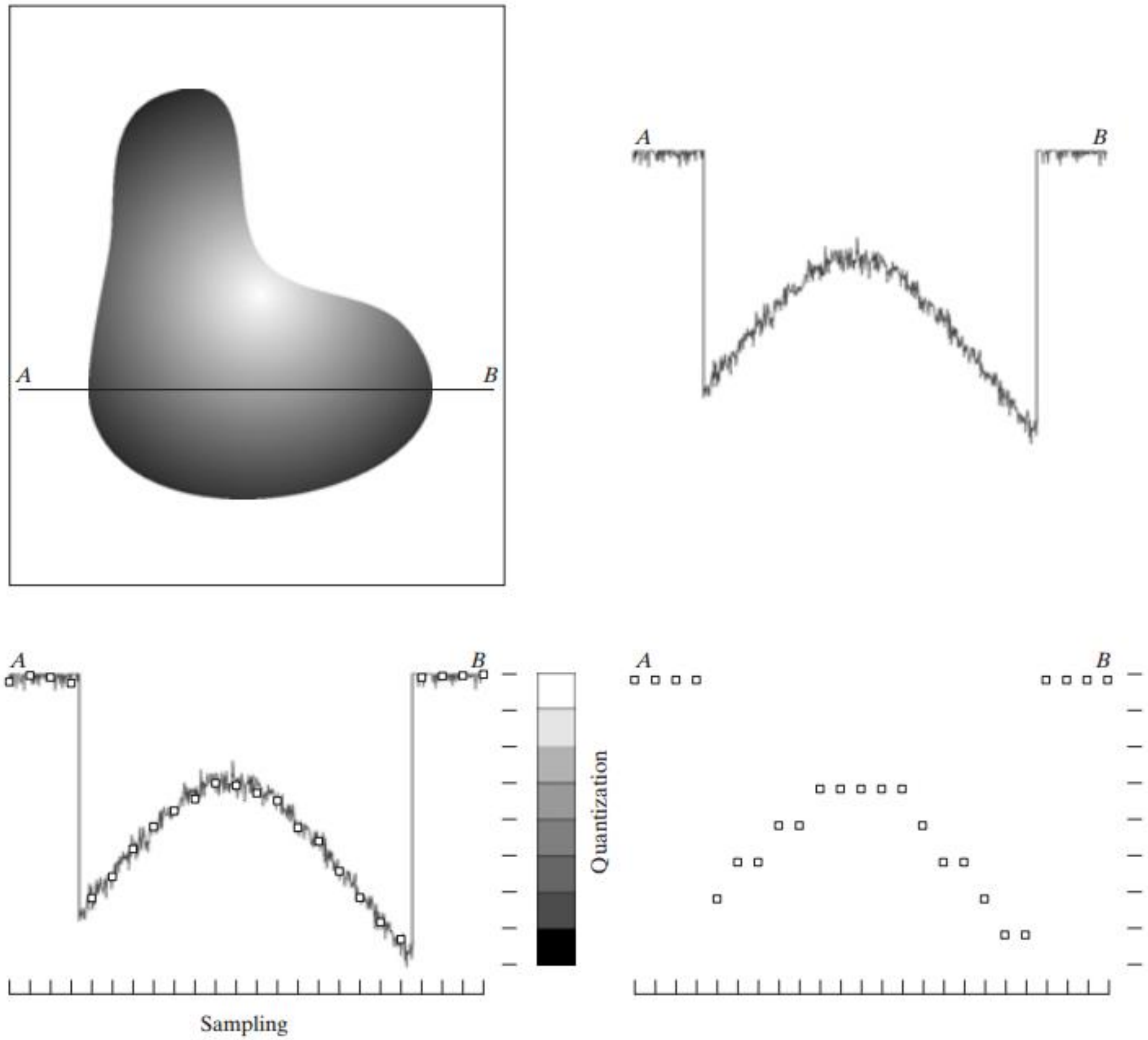
$$L_{\min} \leq \ell \leq L_{\max}$$

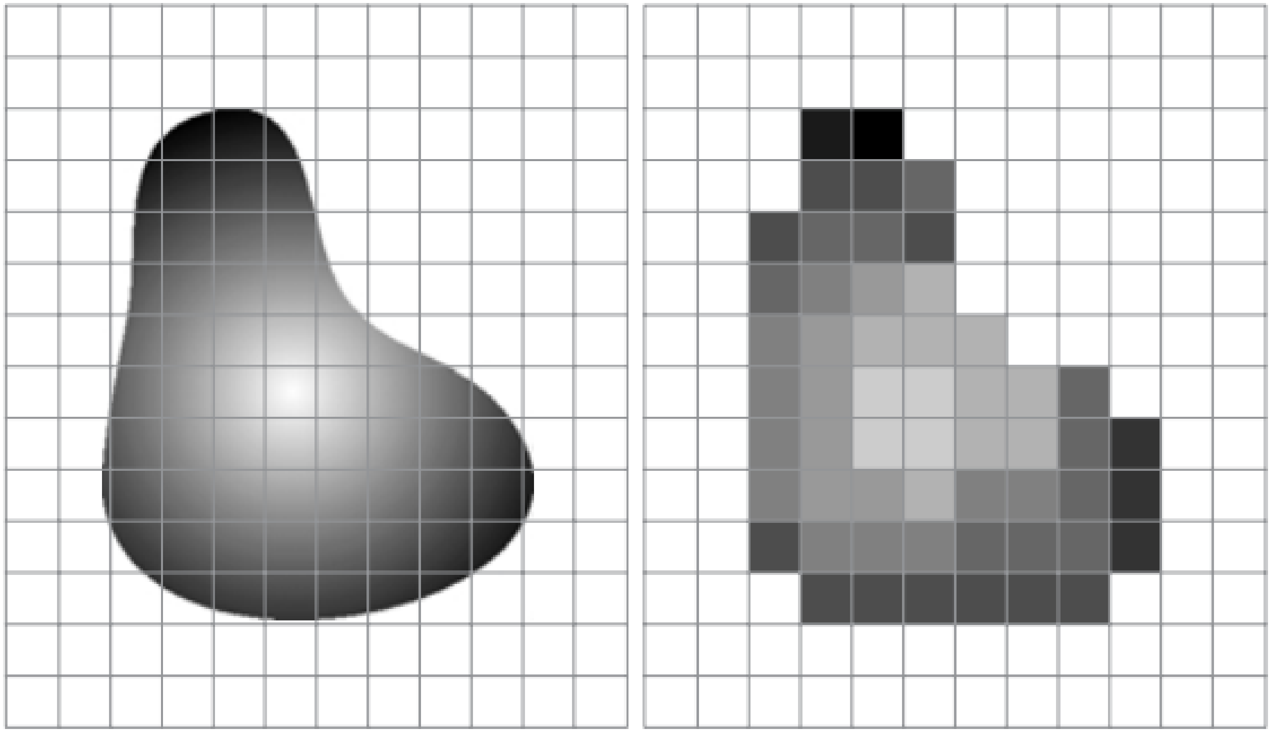
$$L_{\min} = i_{\min} r_{\min} \text{ and } L_{\max} = i_{\max} r_{\max}.$$

The interval $[L_{min}, L_{max}]$ is called the *gray scale*.

Image Sampling and Quantization

- To create a digital image, we need to convert the continuous sensed data into digital form.
- This involves two processes:
  - Sampling : Digitizing the coordinate values is called sampling. The method of sampling is determined by the sensor arrangement used to generate the image.
  - Quantization: Digitizing the amplitude values is called quantization.

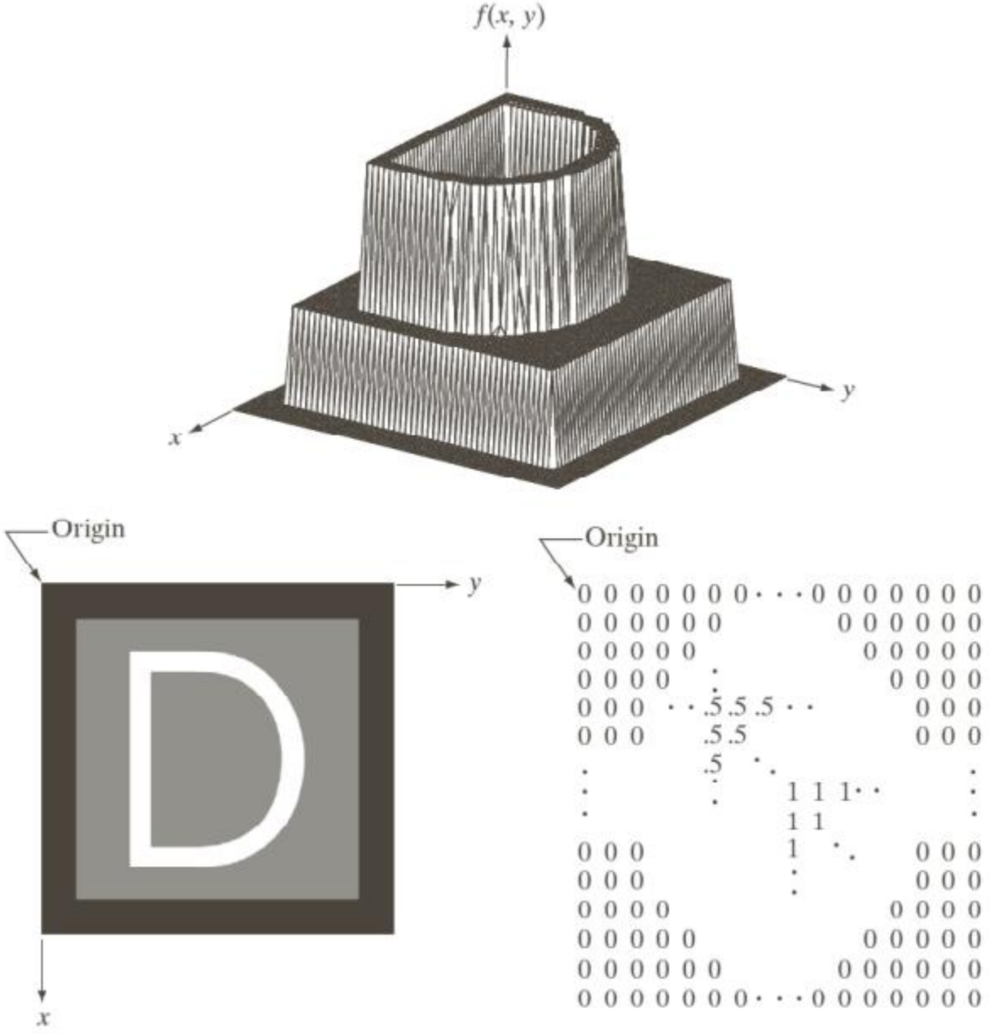

A
B
Quantization
Sampling

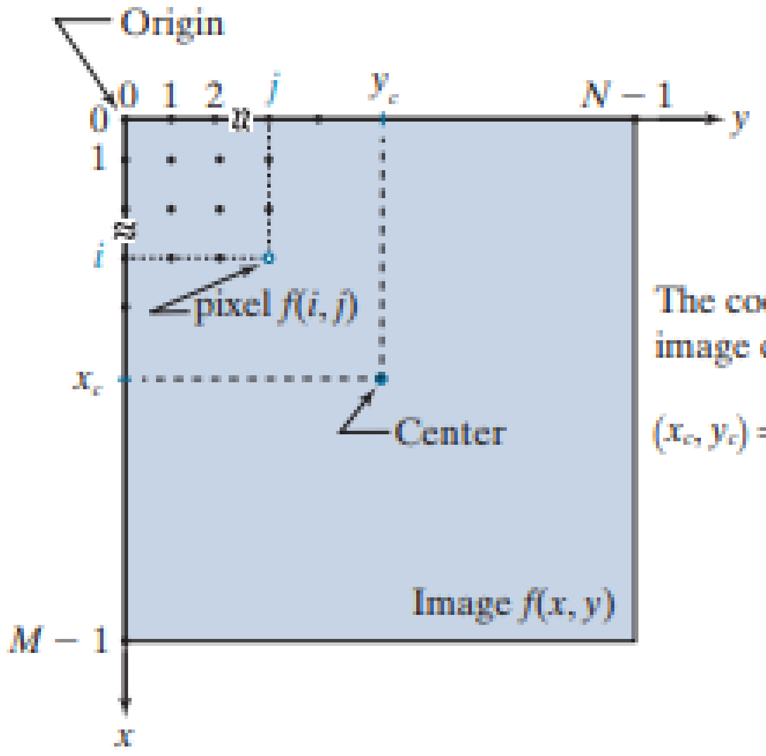


a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Representing digital image





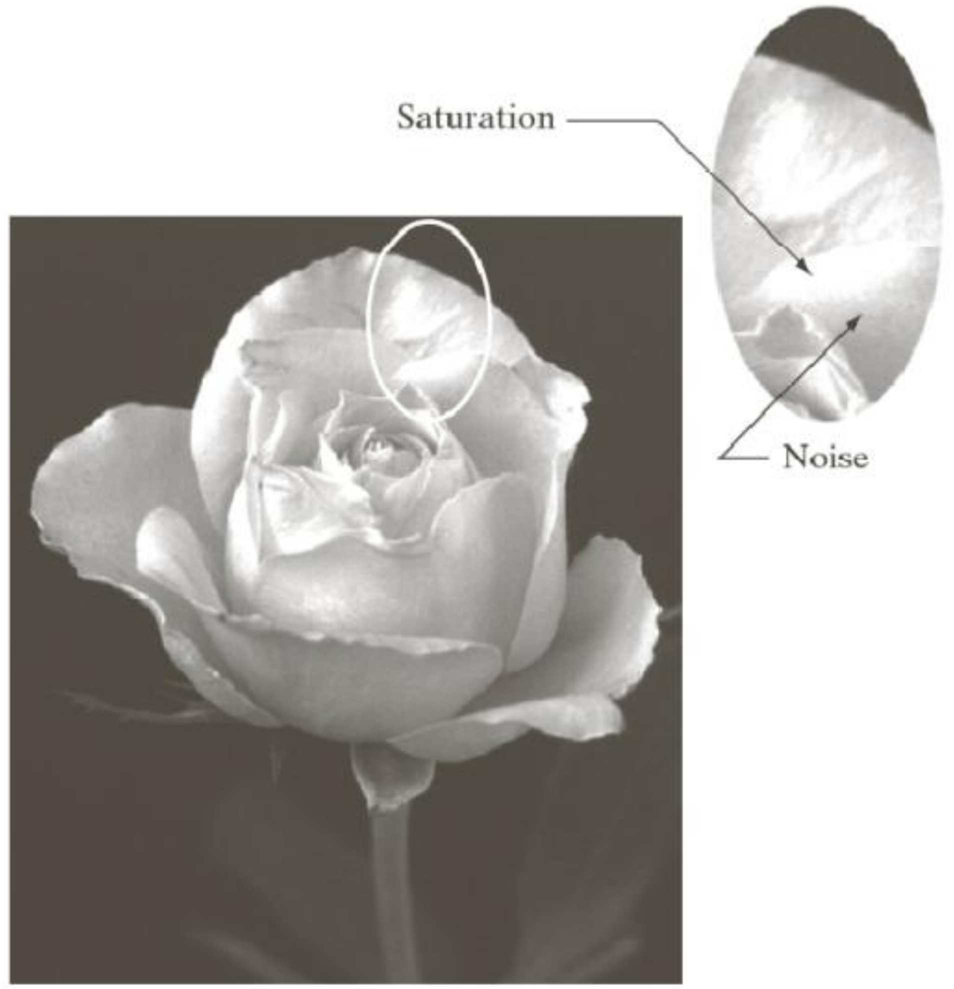
The coordinates of the image center are

$$(x_c, y_c) = \left(\text{floor}\left(\frac{M}{2}\right), \text{floor}\left(\frac{N}{2}\right)\right)$$

Saturation

Saturation is the highest value beyond which all intensity levels are clipped,

the entire saturated area has a high constant intensity level.



SPATIAL AND INTENSITY RESOLUTION

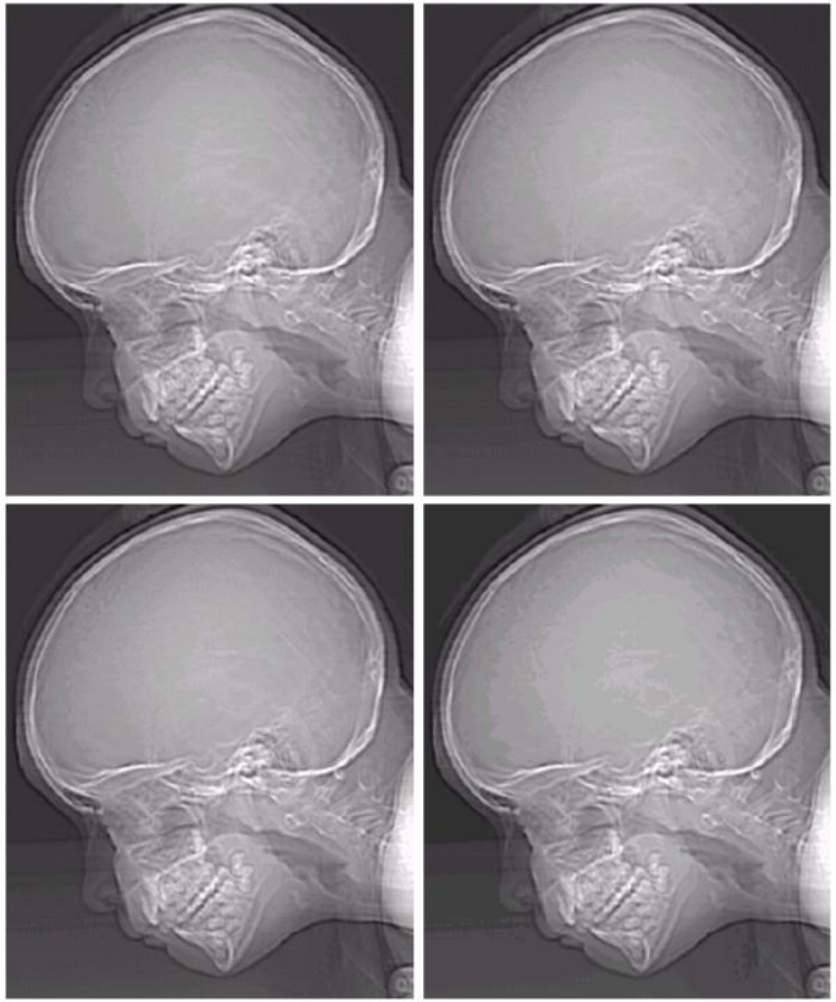
- Spatial resolution:
  - line pairs per unit distance
  - dots (pixels) per unit distance(ex. dots per inch (dpi))
- Intensity resolution:
  - the smallest discernible change in intensity level.
  - the number of bits used to quantize intensity.

Effect of reducing resolution

a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Effect of reducing intensity level

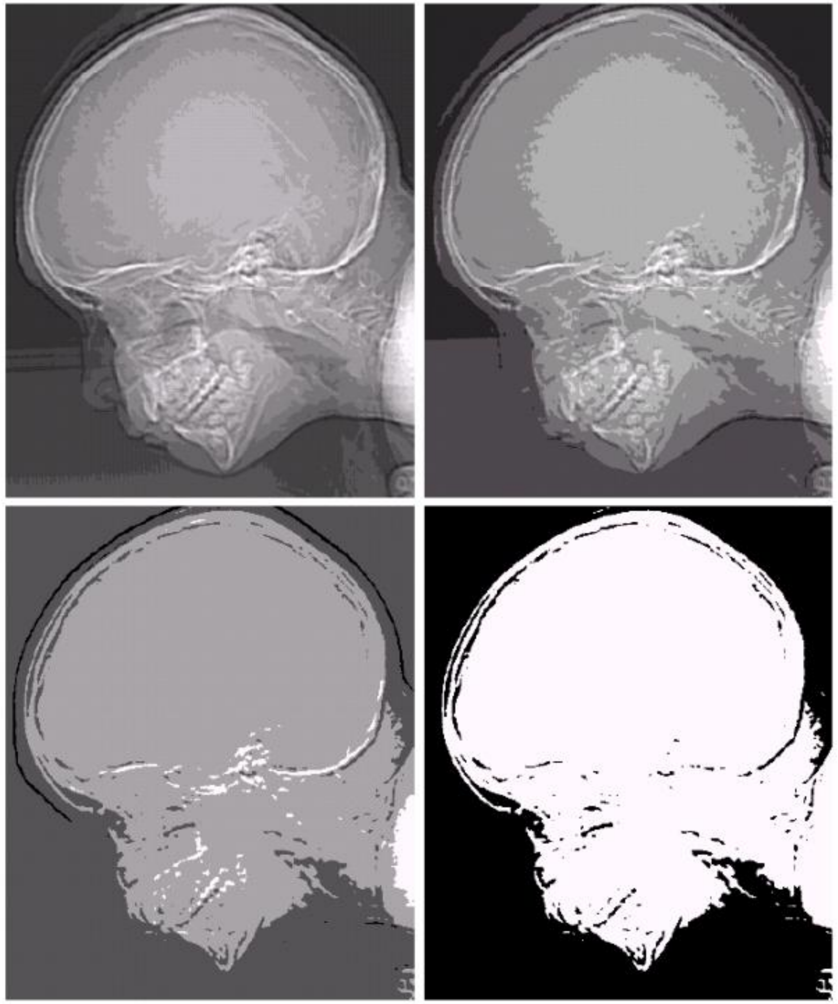


a b
c d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



Effect of reducing intensity level

- The 32-level image has a set of almost imperceptible(human unable to perceive), very fine ridge-like structures in areas of constant intensity. These structures are clearly visible in the 16-level image . This effect, caused by using an insufficient number of intensity levels in smooth areas of a digital image, is called **false contouring**

- reduce no of samples -> distortion effect
- reduce no of gray level-> false contouring

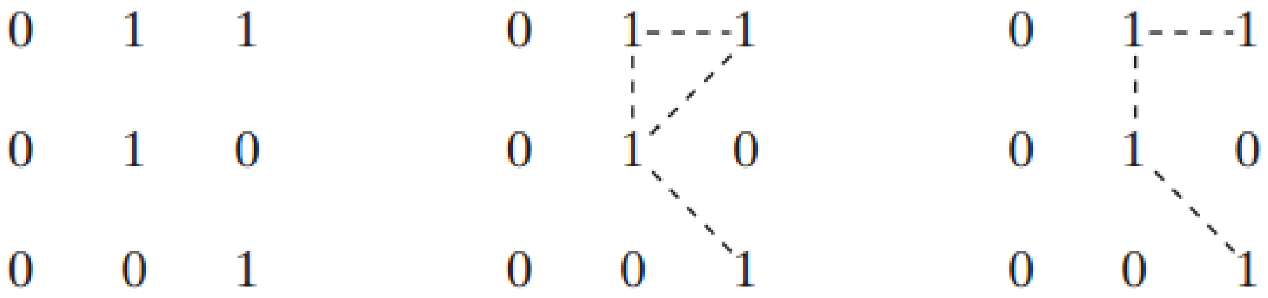
Neighbors of a Pixel

- A pixel p at coordinates (x, y) has *horizontal* and *vertical* neighbors whose coordinates are given by $(x+1, y), (x-1, y), (x, y+1), (x, y-1)$
- This set of pixels, called the 4-*neighbors* of p, is denoted by $N_4(p)$
- The four *diagonal* neighbors of p have coordinates $(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$ and are denoted by $N_D(p)$.
- These points, together with the 4-neighbors, are called the 8-*neighbors* of p, denoted by $N_8(p)$.

Adjacency

- Let *V* be the set of intensity values used to define adjacency. In a binary image, V={1} if we are referring to adjacency of pixels with value 1.
- ***4-adjacency***:Two pixels p and q with values from *V* are 4-adjacent if q is in the set $N_4(p)$.
- ***8-adjacency :*** Two pixels p and q with values from *V* are 8-adjacent if q is in the set N_8(p).
- ***m-adjacency****:* (mixed adjacency).Two pixels p and q with values from *V* are m-adjacent if
  (i) q is in N_4(p), *or*
  (ii) q is in N_D(p) *and* the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from *V*

Adjacency



(a) (b) (c)

(a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel;
(b) (c) m-adjacency.

Path and connectivity

- A *path* (or *curve*) from pixel p with coordinates (x, y) to pixel q with coordinates (s, t) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

$$\text{where } (x_0, y_0) = (x, y), (x_n, y_n) = (s, t),$$

pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for 1 <= i <= n. In this case, n is the *length* of the path.

We can define 4-, 8-, or m-paths depending on the type of adjacency specified.

Path and connectivity

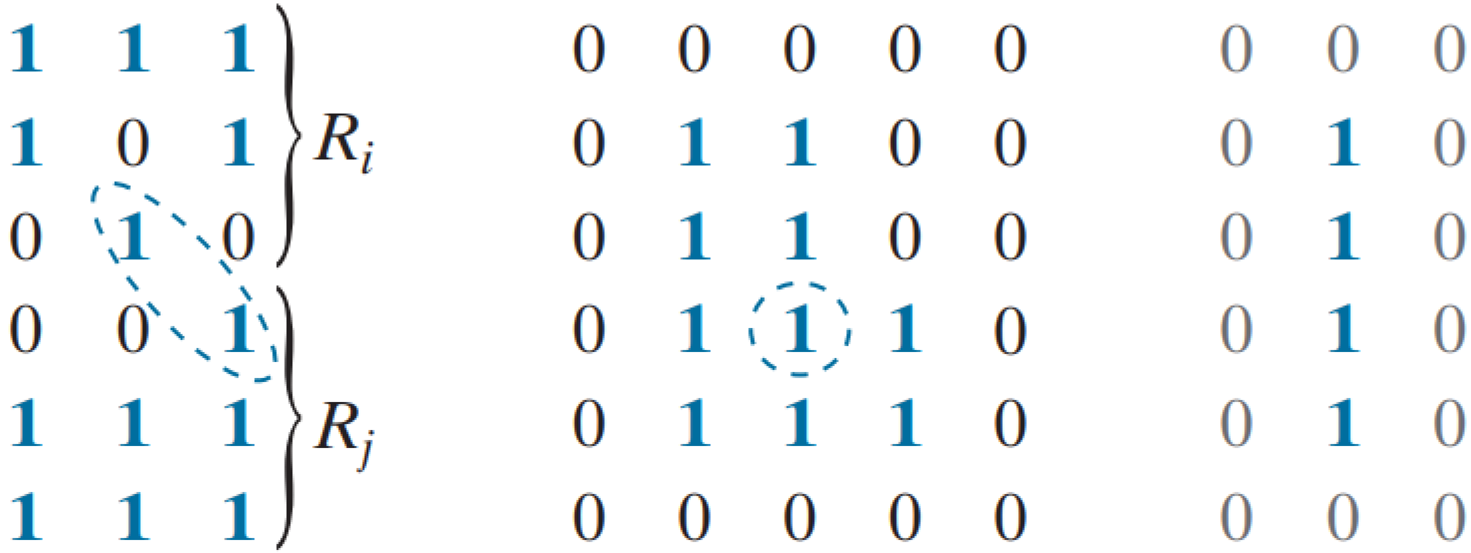
- Let S represent a subset of pixels in an image.
- Two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S.
- For any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S. If it only has one connected component, then set S is called a connected set.

Region and Boundary

- Let R be a subset of pixels in an image. We call R a region of the image if R is a connected set.
- The *boundary* (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*.

- If *R* happens to be an entire image (which we recall is a rectangular set of pixels), then its boundary is defined as the set of pixels in the first and last rows and columns of the image. This extra definition is required because an image has no neighbors beyond its border.

Region and Boundary



ARITHMETIC OPERATIONS

Arithmetic operations between two images $f(x, y)$ and $g(x, y)$ are denoted as

$$
\begin{aligned}
s(x, y) &= f(x, y) + g(x, y) \\
d(x, y) &= f(x, y) - g(x, y) \\
p(x, y) &= f(x, y) \times g(x, y) \\
v(x, y) &= f(x, y) \div g(x, y)
\end{aligned}
$$

Image Averaging

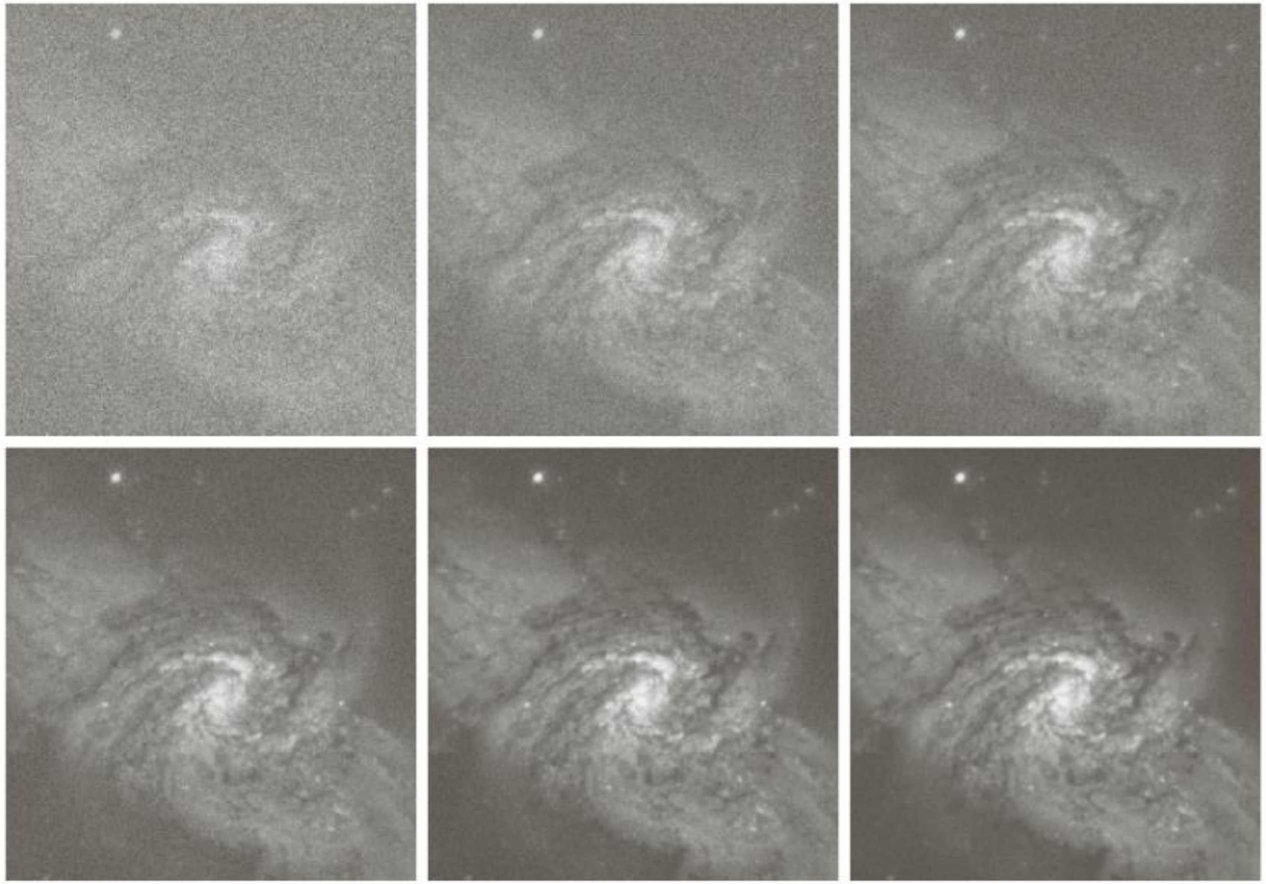


FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

$$g(x, y) = f(x, y) + \eta(x, y)$$

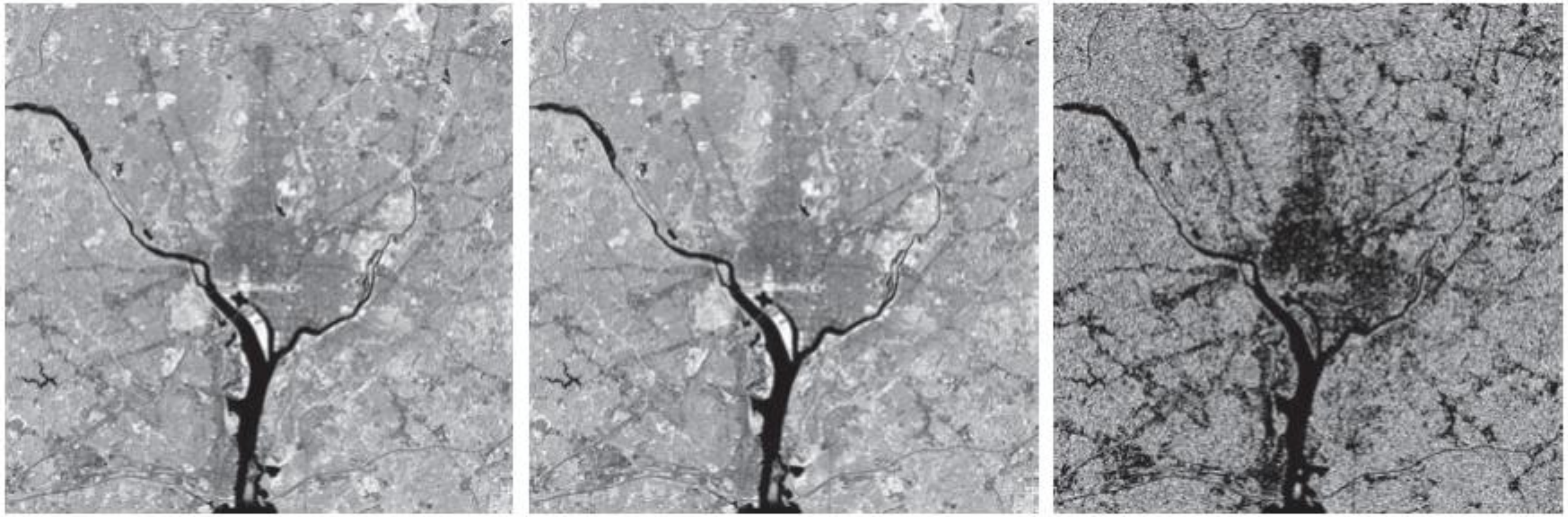
$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

$$E\{\bar{g}(x,y)\} = f(x,y)$$

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$

Image subtraction

- Black (0) values in the difference image indicate locations where there is no difference between the images in Figs. 2.30(a) and (b).



a b c

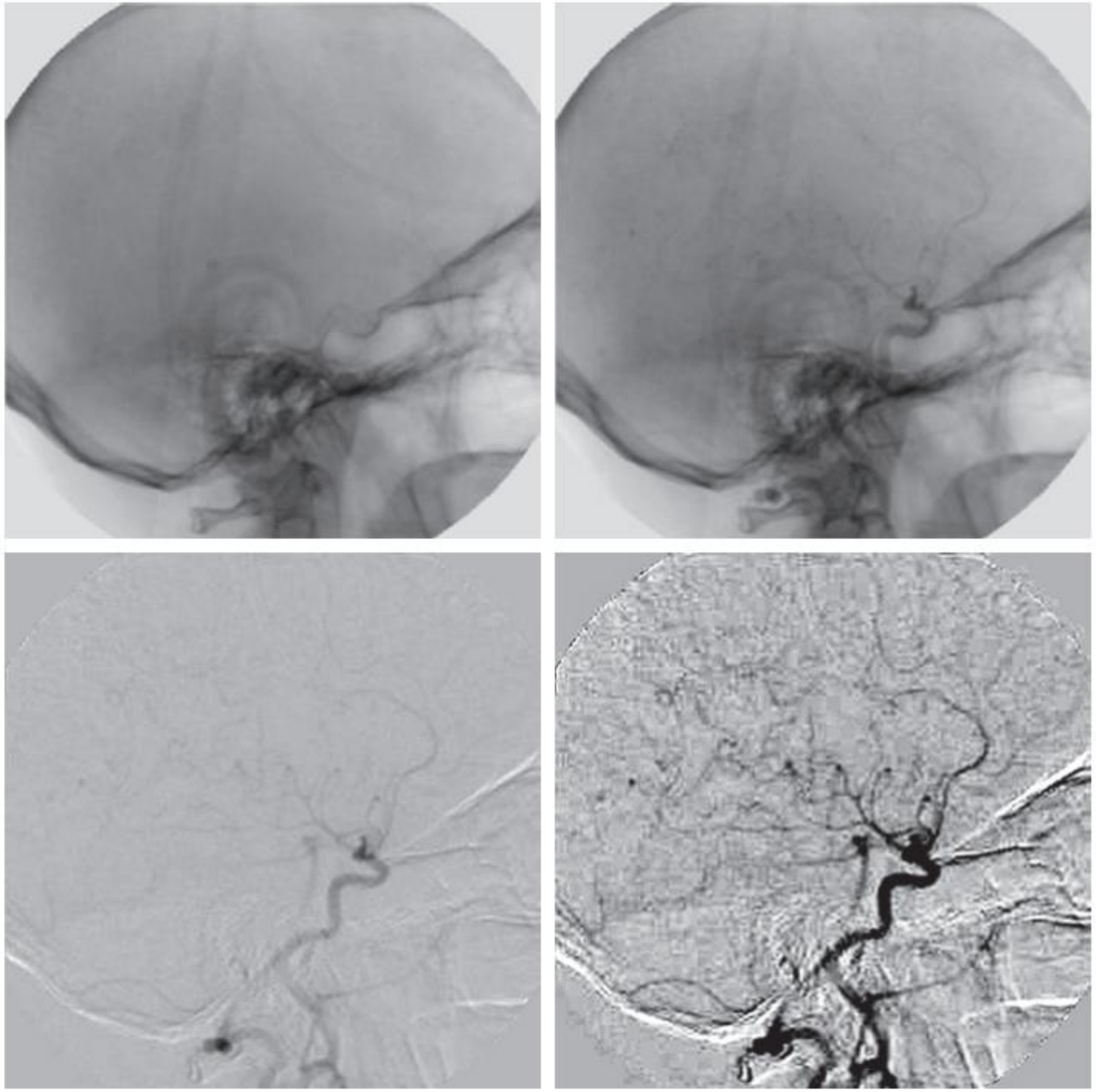
FIGURE 2.30 (a) Infrared image of the Washington, D.C. area. (b) Image resulting from setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity. (Original image courtesy of NASA.)

Mask Mode Radiography

$g(x,y) = f(x,y) - h(x,y)$

$h(x,y)$ = the mask, X-ray image of a region of a patient's body

$f(x,y)$ = live image [injecting an X-ray contrasted Medium into patient's bloodstream, and taking a series of images called live images]



a b
c d

FIGURE 2.32 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of the Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

Shading correction

$g(x,y) = f(x,y) * h(x,y)$

h(x,y) = shading function
f(x,y) = perfect image

a b c

FIGURE 2.33 Shading correction. (a) Shaded test pattern. (b) Estimated shading pattern. (c) Product of (a) by the reciprocal of (b). (See Section 3.5 for a discussion of how (b) was estimated.)

SET operations

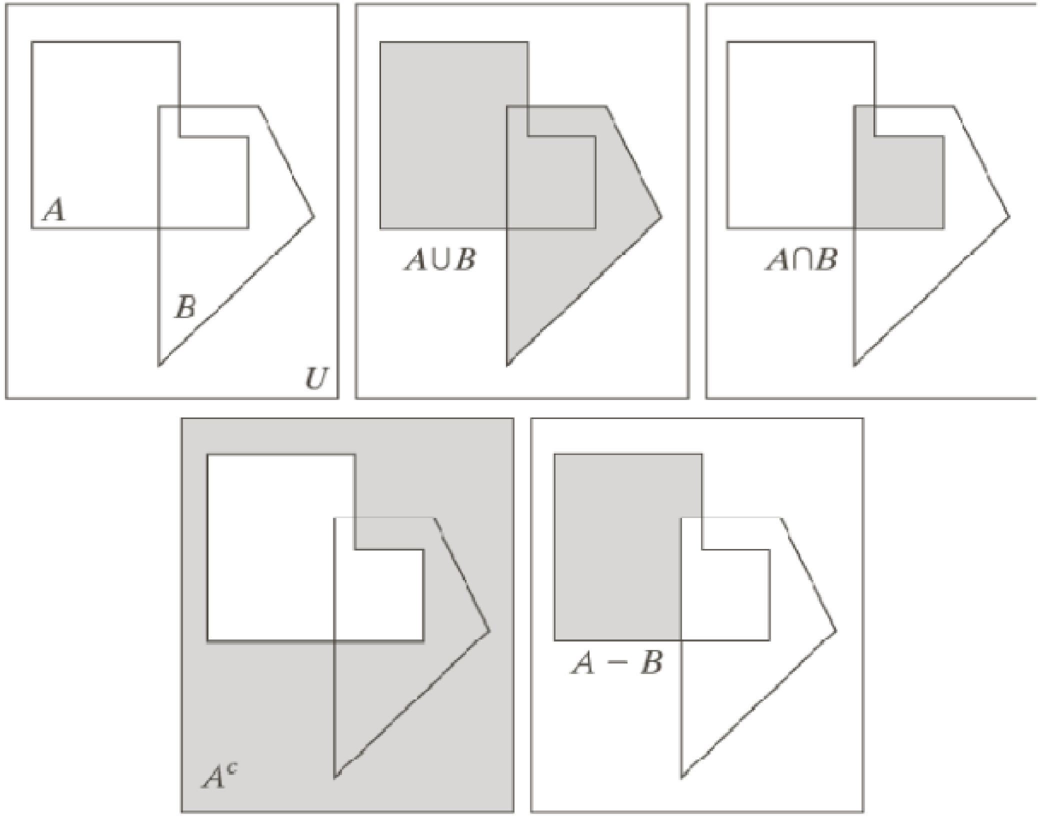


FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

SET operations

- A set is denoted by the contents of two braces: { }

$$C = \{c \mid c = -d,\ d \in D\}$$

$$D = A \cap B \qquad A \cup B = \{\max_z(a, b) \mid a \in A,\ b \in B\}$$

$$C = A \cup B$$

The *complement* of a set A is the set of elements that are not in A:

$$A^c = \{w \mid w \notin A\} \qquad A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

The *difference* of two sets A and B, denoted $A - B$, is defined as

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

the set difference operation; that is, $A^c = \Omega - A$.

SET operations

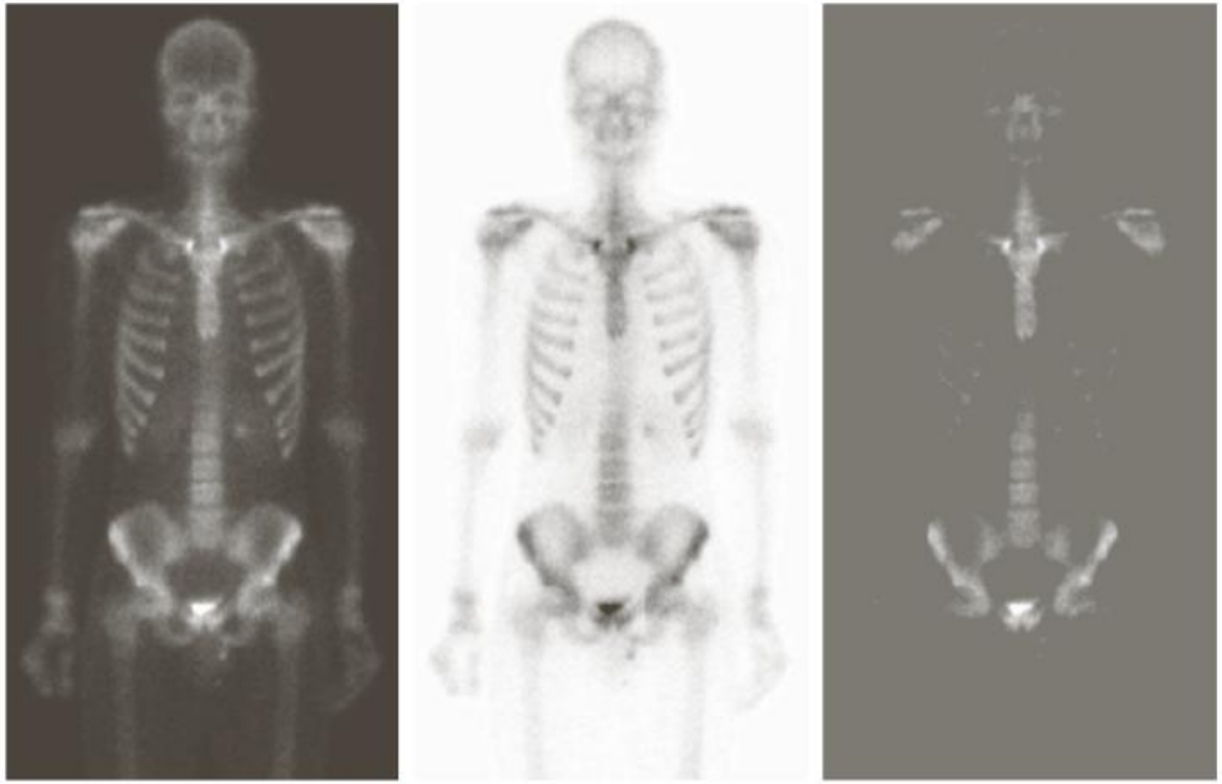


FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Averaging using neighborhood

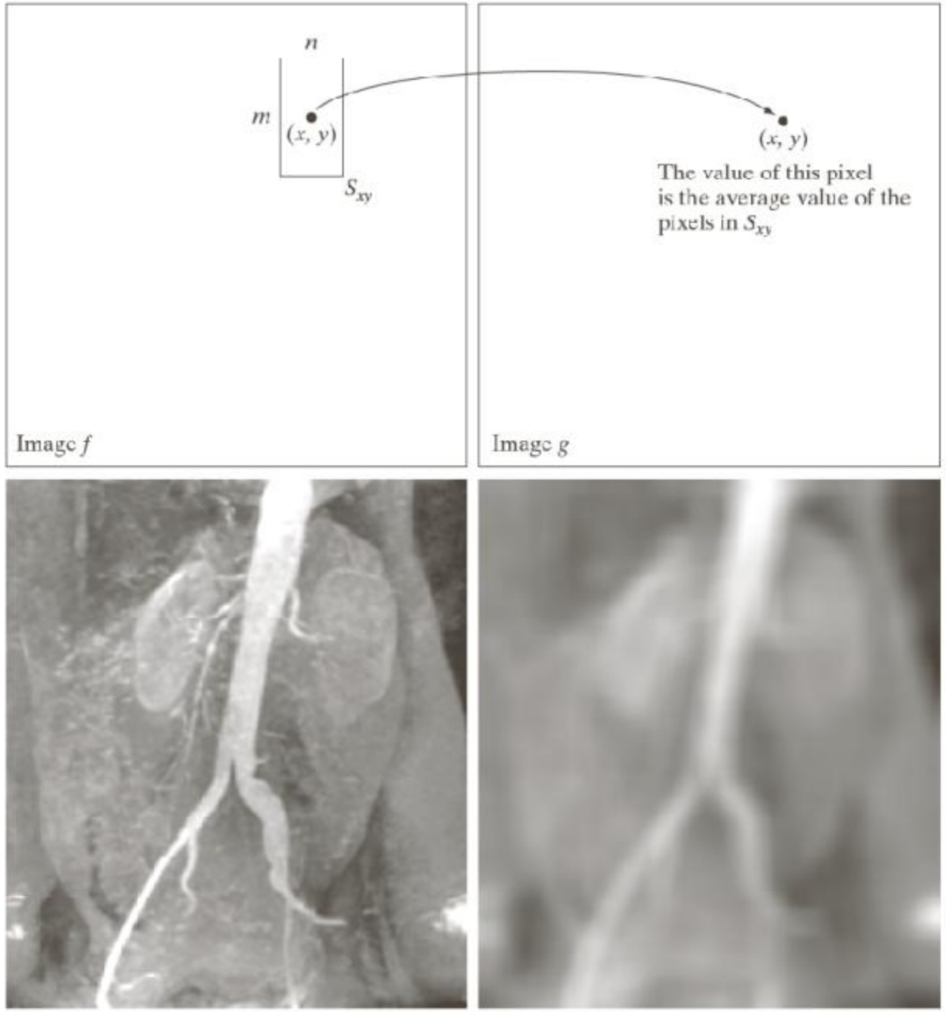


FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.

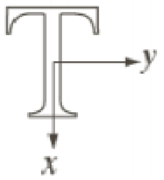
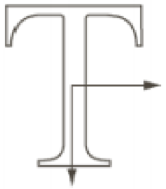
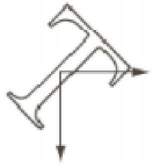
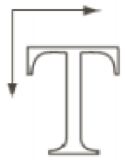
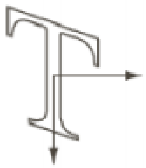
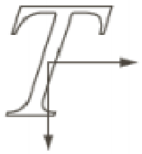
Transformation

- The transformation of co-ordinates may be expressed as

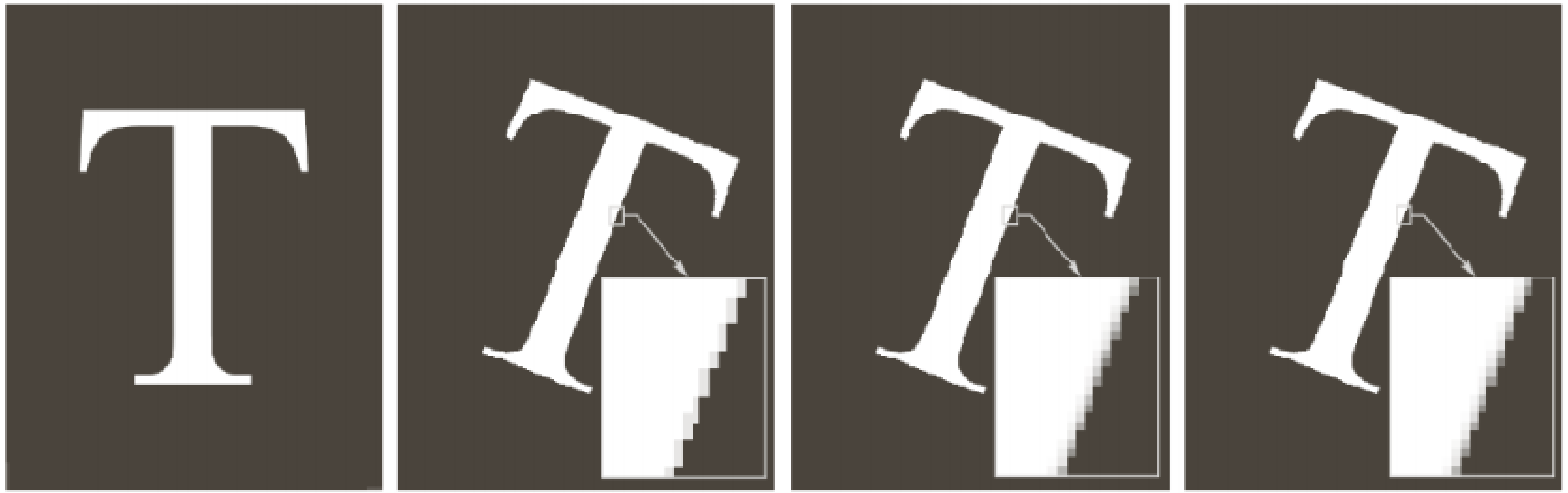
$$(x,y) = T\{(v,w)\}$$

(v,w) = pixel co-ordinates in the original image

(x,y) = corresponding pixel co-ordinates in the transformed image

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Rotation

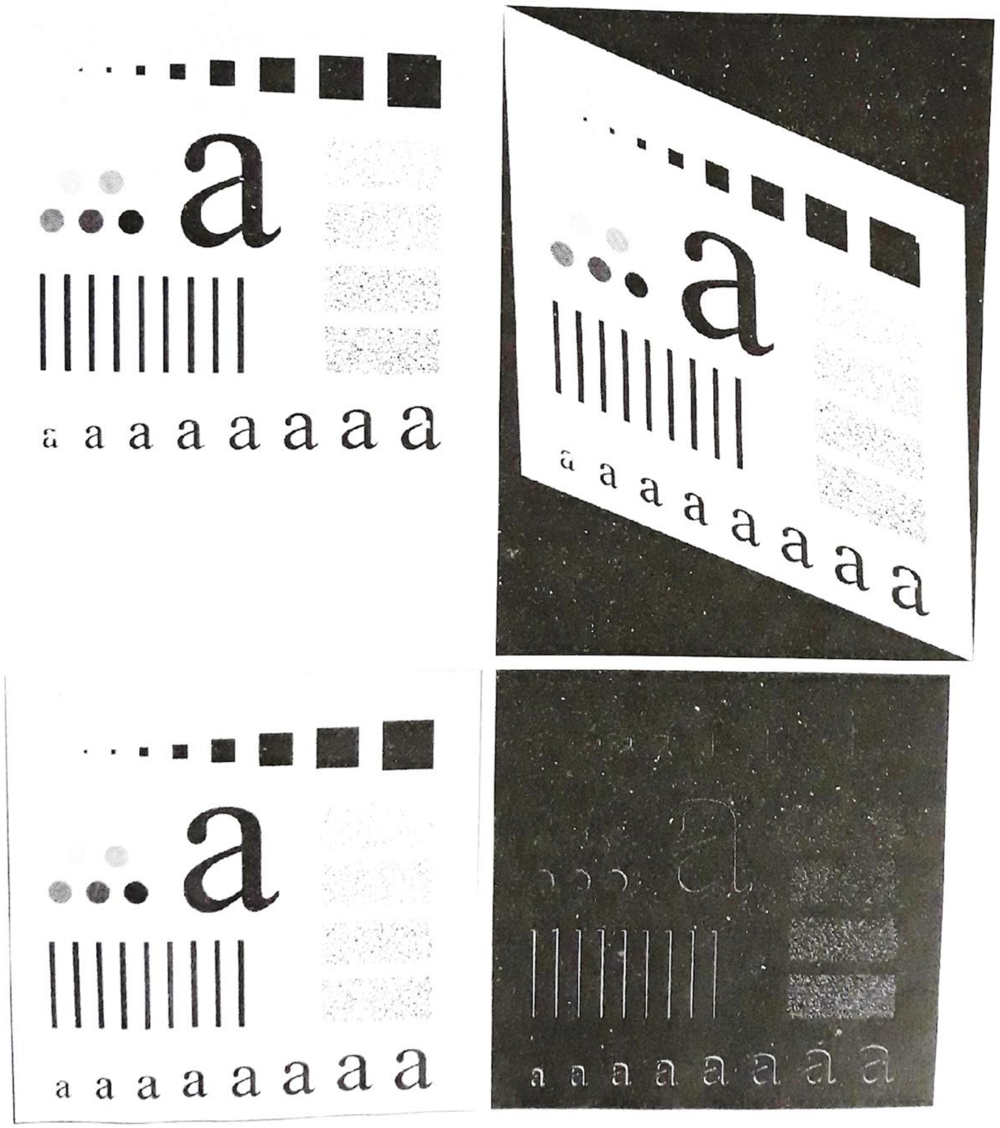


a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Image registration

- Used to align two or more images
- Available:
  - input and output images
- Specific transformation is unknown



a b
c d

FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the border). (d) Difference between (a) and (c), showing more registration errors.

End